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# SENSITIVITY BEHAVIOR ANALYSIS IN DISTRIBUTED PARAMETER ESTIMATION

**Chang-Bock Chung**

Department of Fine Chemical Engineering, Chonnam National University,
Yongbong-dong 300, Buk-ku, Kwangju 500-757, Korea
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**Abstract**—This paper is concerned with sensitivity analysis for the distributed parameter estimation problem arising in the modeling process for fluid flow in underground porous media. An efficient algorithm was constructed using variational calculus techniques for the evaluation of sensitivity gradient curves which describe variations of system outputs resulting from variations of a spatially-varying parameter in nonlinear partial differential equations. For a test problem of estimating transmissivity of a one-dimensional ideal gas reservoir, sensitivity behavior was analyzed under various reservoir conditions, and the results were applied to devising a parameter discretization scheme which will yield improved parameter estimates.

## INTRODUCTION

The problem of estimating spatially varying parameters in distributed parameter systems (DPS) arises in many areas of science and engineering. The present work has been primarily motivated by the modeling of fluid flow in underground porous media, such as petroleum reservoirs or aquifers. The spatially varying properties to be estimated represent unknown reservoir parameters such as permeability and porosity. These parameters are inaccessible to direct measurement, and, therefore, have to be estimated on the basis of measured pressure and flow rate histories; this estimation process is commonly referred to as history-matching [1, 2].

The major difficulty in developing successful solution techniques lies in the ill-posedness of the problem in the sense that small errors in the data may cause large errors in the estimates (*instability*), or the given data may not provide enough information to determine a unique estimate (*unidentifiability*) [3]. This ill-posedness is directly related to the behavior of sensitivity which is defined to be the partial derivative of the output of a system with respect to the parameter. Roughly speaking, instability is ascribed to a very small sensitivity, while unidentifiability to a vanishing sensitivity. Thus, sensitivity analysis has been undertaken in many studies on parameter estimation to understand the ill-posed behavior and thereby to develop efficient solution algorithms [4, 5].

In lumped parameter systems (LPS) where parameters take on constant values, sensitivity coefficients can be easily evaluated, and then sensitivity analysis centers around the analysis of the sensitivity coefficient matrix. On the other hand, in distributed parameter systems where the parameter is given as a function of the spatial variable, sensitivity takes the form of a functional gradient, which we will call a *sensitivity gradient* in the following. It delivers the information on how the output will change due to the regional variation of the parameter.

But, despite the detailed information it delivers, sensitivity gradient has been neglected in most studies because of *a priori* discretization of parameters adopted in constructing numerical estimation algorithms. It is a common practice to represent the unknown parameter as a linear combination of shape functions, e.g. B-splines, and then to estimate the spline coefficients; this effectively reduces a DPS estimation problem to an LPS one. However, the resulting sensitivity coefficients may only represent lumped information along the profiles of the shape functions, hence lacking spatial details.

This paper focuses on the analysis of sensitivity gradient arising in a DPS estimation problem. First, an estimation problem and its solution algorithms are presented. Then, an optimal control formulation is presented for the evaluation of the sensitivity gradient. Finally, the algorithm is applied to a test problem of estimating transmissivity of a one-dimensional ideal gas reservoir.

## A DISTRIBUTED PARAMETER ESTIMATION PROBLEM AND ITS SOLUTION ALGORITHMS

The pressure distribution u(x, t) of fluid flowing in porous media is governed by the following nonlinear partial differential equation [6].

$$\frac{\partial u}{\partial t} = \nabla \cdot [\alpha(x)\ \phi(u)\ \nabla u] + q(x, t),\ \text{in}\ \Omega \times (0, T) \quad (1)$$

subject to initial and no-flow boundary conditions

I.C. $u(x, 0) = u_0(x)$ in $\Omega$

B.C. $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega \times (0, T)$

In the above equation, $\alpha(x)$ denotes the transmissivity of media which is a measure of ease for fluid flow in the domain $\Omega$, $\phi(u)$ a property-dependent term, q(x, t) the withdrawal or injection of fluid, n the outward unit normal vector to the boundary $\partial\Omega$.

A parameter estimation problem associated with the model Eq. (1) can be described as follows.

Knowing the initial pressure distribution $u_0(x)$ and given a set of measurements of pressure $\{u^{obs}(x_i, t_k);\ i=1,\cdots, n_{ow},\ k=1,\cdots, n_t\}$ and a production rate q(x, t) up to time T, determine the spatially varying parameter $\alpha(x)$.

Solution algorithms to the above estimation problem are commonly constructed on the basis of nonlinear regression in the following three steps [7].

**1. Formulation Step**

The estimation problem is formulated as a nonlinear optimization problem of minimizing an objective functional. In most cases where the output data are the only information available on the system, a least-squares functional is regarded as the most "natural" objective functional.

$$\min_{\alpha} J_{LS}(\alpha) = \sum_{i=1}^{n_{ow}} \sum_{k=1}^{n_t} [u(x_i, t_k;\ \alpha) - u_{ik}^{obs}]^2 \quad (2)$$

Depending on the availability of valid statistical assumptions on observation errors or *a priori* information on the parameter, one may use a modified objective functional [8].

**2. Discretization Step**

This step provides a computational framework for minimization of the objective functional. Specifically, (2) is converted into an approximate finite dimensional minimization problem on the basis of two kinds of discretization: (i) finite difference or finite element solution of the model equation (*state discretization*) and (ii) representation of the spatially varying parameter as a linear combination of shape functions (*parameter discretization*). For example, the parameter can be approximated using B-splines as follows [9].

$$\alpha(x) = \sum_{j=1}^{N_p} \omega_j\ B_j(x) \quad (3)$$

**3. Optimization Step**

In this step the discretized objective functional is actually minimized with respect to the spline coefficients introduced in (3).

$$\min_{\omega} J_{LS}(\omega),\ \omega \in R^{N_p} \quad (4)$$

Typically the minimization is carried out using an iterative scheme of the following form

$$\omega^{(k+1)} = \omega^{(k)} + \gamma^{(k)}\ d^{(k)}, \quad (5)$$

where $d^{(k)}$ denotes the descent direction at the k-th major iteration, and $\gamma^{(k)}$ a step length along the descent direction.

## SENSITIVITY EVALUATION ALGORITHMS

Sensitivity is defined as the partial derivative of a system output with respect to a model parameter. The system output associated with the parameter estimation problem stated in the previous section consists of point values $u(x_i, t_k;\ \alpha)$, $i=1,\cdots, n_{ow}$, $k=1,\cdots, n_t$ (abbreviated as $u_{ik}$). $\alpha$ being a function of the spatial variable x, each sensitivity $\partial u_{ik}/\partial\alpha$ takes the form of a functional gradient, whose meaning will be made clear in the following.

In functional analysis, a gradient of a functional $J(\alpha)$ defined on a Hilbert space H is the unique element $\varphi$ that satisfies

$$\delta J = (\varphi, \delta\alpha)_H \text{ for every } \delta\alpha \in H, \quad (6)$$

where $\delta$ denotes the variational operator, $(\cdot, \cdot)_H$ the inner product in the Hilbert space [10]. With the aid of delta functions, the output $u_{ik}$ can be represented as a functional defined on $L_2(\Omega)$.

$$J(\alpha) = u(x_i,\ t_k;\ \alpha)$$
$$= \int_0^T \int_\Omega u(x, t;\ \alpha)\ \delta(x - x_i)\ \delta(t - t_k)\ dx\ dt \quad (7)$$

Thus, the Eq. (6) with $L_2(\Omega)$ chosen for H implies that when multiplied by a variation of parameter and integrated over the domain, the sensitivity gradient $\partial u_{ik}/\partial\alpha$ would give the consequent change in the model output $u_{ik}$. In other words, the profile of $\partial u_{ik}/\partial\alpha$

represents the influence of regional variation of the parameter on the output.

The gradient $\partial u_{ik}/\partial\alpha$ satisfying (6) was found using an optimal control formulation for the objective functional given in (7). Here the parameter $\alpha$ is regarded as a *control* variable that minimizes the objective functional $J(\alpha)$ under the constraint of the state Eq. (1). First, an augmented objective functional is constructed by introducing an adjoint variable $\lambda(x, t)$.

$$J(\alpha)=\int_0^T\int_\Omega\Big[u(x,t;\ \alpha)\ \delta(x-x_i)\ \delta(t-t_k) + \lambda(x,t)\Big\{\frac{\partial u}{\partial t}-\nabla\cdot[\alpha\ \phi(u)\nabla u]-q\Big\}\Big]dx\,dt \quad (8)$$

The first variation of $J(\alpha)$ is expanded as

$$\delta J=\int_0^T\int_\Omega\Big[\delta(x-x_i)\ \delta(t-t_k)\ \delta u+\lambda\Big\{\frac{\partial\delta u}{\partial t} -\nabla\cdot[\alpha\,\phi'(u)\delta u\ \nabla u]-\nabla\cdot[\alpha\,\phi(u)\,\nabla\delta u]\Big\} +\lambda\ \nabla\cdot[\delta\alpha\ \phi(u)\ \nabla u]\Big]dx\,dt \quad (9)$$

Next, applying the Green's formula and the identity equation

$$\nabla\cdot\{\alpha\,\phi'(u)\delta u\ \nabla u\}+\nabla\cdot\{\alpha\,\phi(u)\ \nabla\delta u\} =\nabla\cdot[\alpha\ \nabla\{\phi(u)\delta u\}], \quad (10)$$

(9) is arranged to give

$$\delta J=\int_0^T\int_\Omega\Big[\Big\{-\frac{\partial\lambda}{\partial t}-\phi(u)\ \nabla\cdot(\alpha\ \nabla\lambda)+\delta(x-x_i)\ \delta(t-t_k)\Big\}\ \delta u\ dx\,dt+\int_\Omega\lambda\ \delta u\Big]_{t=T}dx +\int_0^T\int_{\partial\Omega}\alpha\ \frac{\partial\lambda}{\partial n}\ \phi(u)\delta u\ ds\,dt +\int_\Omega\Big\{\int_0^T\phi(u)\ \nabla u\cdot\nabla\lambda\ dt\Big\}\ \delta\alpha\ dx \quad (11)$$

Finally, letting the terms on the right hand side vanish except the last one, one can obtain

$$\frac{\partial J}{\partial\alpha}=\frac{\partial u_{ik}}{\partial\alpha}=\int_0^T\phi(u)\ \nabla u\cdot\nabla\lambda\ dt \quad (12)$$

where u satisfies the state Eq. (1), and $\lambda$ satisfies the adjoint equation,

$$\frac{\partial\lambda}{\partial t}=-\phi(u)\ \nabla\cdot\{\alpha(x)\ \nabla\lambda\}+\delta(x-x_i)\ \delta(t-t_k), \quad \text{in } \Omega\times(0,T) \quad (13)$$

I.C.: $\lambda(x, T)=0$ in $\Omega$

B.C.: $\dfrac{\partial\lambda}{\partial n}=0$ on $\partial\Omega\times(0,T)$

When a parameter to be estimated is discretized *a priori* as in (3), it is sensitivity coefficients, e.g., $\partial u_{ik}/\partial\omega_j$, $j=1,\cdots, N_p$ that count in actual numerical implementation. Once a sensitivity gradient is obtained, each sensitivity coefficient can be calculated as follows.

$$\frac{\partial u_{ik}}{\partial\omega_j}=\int_\Omega\frac{\partial u_{ik}}{\partial\alpha}\ B_j(x)\ dx \quad (14)$$

However, it should be mentioned that the integral operation of (14) implies the lumping of spatial details of sensitivity behavior along the profile of the shape function $B_j(x)$; this may result in ill-conditioning in the optimization step unless the parameter is properly discretized.

## NUMERICAL EXPERIMENTS

### 1. Test Problem

A one-dimensional ideal gas reservoir {where $\phi(u)=u/\mu$} was considered as a test problem to demonstrate the algorithms described so far. For an easy grasp of the relative magnitude of numerical values involved, the governing equation was converted into a dimensionless form by scaling each variable or parameter to the following reference values, respectively: reservoir length L=10 Km, total time lapse T=365 days, pressure $u_0=200$ atm, total production rate $q_t=0.219$ atm/day, and transmissivity $\alpha_0=6.58\times10^{-7}$ $m^2$.

$$\frac{\partial u}{\partial t}=\kappa\frac{\partial}{\partial x}\Big\{\alpha(x)\ u\frac{\partial u}{\partial x}\Big\}+\eta\sum_{w=1}^{npw}q_w\ \delta(x-x_w), \quad \text{in } (0,\ 1)\times(0,\ 1) \quad (15)$$

I.C. $u(x,\ 0)=u_0(x)$ in (0, 1)

B.C. $\dfrac{\partial u}{\partial x}(0,\ t)=\dfrac{\partial u}{\partial x}(1,\ t)=0$ in (0, 1)

where $\kappa=\alpha_0Tu_0/\mu L^2=0.2365$

$\eta=q_tT/u_0=0.4$

Similarly, the corresponding adjoint equation and sensitivities were obtained in dimensionless forms.

$$\frac{\partial\lambda}{\partial t}=-\kappa u\frac{\partial}{\partial x}\Big\{\alpha(x)\ \frac{\partial\lambda}{\partial x}\Big\}+\delta(x-x_i)\delta(t-t_k), \quad \text{in } (0,\ 1)\times(0,\ 1) \quad (16)$$

I.C. $\lambda(x,\ 1)=0$ in (0, 1)

B.C. $\dfrac{\partial\lambda}{\partial x}(0,\ t)=\dfrac{\partial\lambda}{\partial x}(1,\ t)=0$ in (0, 1)

$$\frac{\partial u_{ik}}{\partial\alpha}=\kappa\int_0^1u\ \frac{\partial u}{\partial x}\ \frac{\partial\lambda}{\partial x}\ dt \quad (17)$$

$$\frac{\partial u_{ik}}{\partial\omega_j}=\int_0^1\frac{\partial u_{ik}}{\partial\alpha}\ B_j(x)\ dx \quad (18)$$

## 2. Numerical Algorithms

Numerical solutions to the state Eq. (15) were obtained using a finite difference method (FDM). The spatial domain (0, 1) was divided into 100 uniform intervals. 101 point-centered grids were used to approximate the state variable, while 100 block-centered grids for the parameter. The spatial derivative term in (15) was converted first,

$$u\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial u^2}{\partial x}, \qquad (19)$$

and then was approximated using central difference formula. The time derivative was approximated using backward difference. The resulting system of nonlinear equations were solved using Newton-Raphson iterations.

Similarly, the adjoint Eq. (16) was solved using FDM on the same mesh. The linearity of Eq. (16) in $\lambda$ exempts one from Newton-Raphson iterations in solving finite difference equations. The time steps march backward from $t=1$ on. Due to the Neumann type boundary condition, however, the initial relaxed state at $t=1$ persists until $t_k$. So the computation time could be reduced by starting from $t_k$.

Once u and $\lambda$ were obtained, the value of sensitivity gradient at the m-th parameter grid was evaluated by approximating the derivatives by finite difference

$$\left(\frac{\partial u_{ik}}{\partial \alpha}\right)_m \simeq \frac{\kappa}{2(\Delta x)^2}\int_0^1 (u_{j+1}^2 - u_j^2)\,(\lambda_{j+1}-\lambda_j)\,dt, \qquad (20)$$

and then using the Simpson's rule. The sensitivity coefficient with respect to the j-th spline coefficient is also calculated as follows.

$$\frac{\partial u_{ik}}{\partial \omega_j} \simeq \sum_{m\in I_{pj}} \left(\frac{\partial u_{ik}}{\partial \alpha}\right)_m B_j(x_m)\,\Delta x \qquad (21)$$

where $I_{pj}=\{m:\ B_j(x_m)\neq 0\}$

Estimation of $\alpha(x)$ proceeded in three steps as described previously. First, a least squares objective functional was constructed as

$$\min_{\alpha} J_{LS}(\alpha) = \sum_{i=1}^{n_{ow}} \sum_{k=1}^{nt} \{u(x_i, t_k;\ \alpha) - u_{ik}^{obs}\}^2 \qquad (22)$$

The observation data were simulated by adding to the calculated values of u random numbers having normal distribution with mean zero and a standard deviation of 0.01. Next, the parameter is discretized using B-splines. Finally, the discretized objective functional was minimized using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) quasi-Newton algorithm [11] and the golden section method for line search.

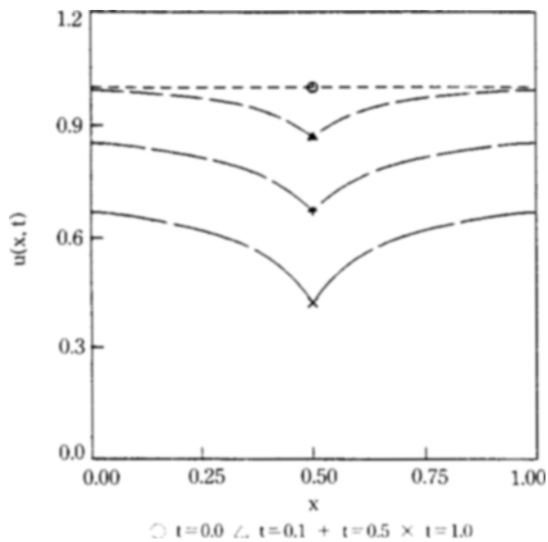


**Fig. 1. u(x, t) vs. x for $\alpha(x)=1.0$ with $n_{pw}=1$ at $x_w=0.5$.**

# RESULTS AND DISCUSSION

## 1. State Behavior

Fig. 1 shows typical pressure distributions in a reservoir which is assumed to have a uniform parameter $\alpha(x)=1$ and a single production well located in the middle region. The initial uniform distribution can be seen to move toward the excavated profiles as fluid is withdrawn from the well. Indeed it is the data gathered from this "excited" system that forms the ground of parameter estimation. The assumed symmetry in reservoir conditions is reflected in the symmetry of the profiles.

## 2. Adjoint Behavior

Fig. 2 shows typical behavior of the dimensionless adjoint variable obtained in the process of evaluating a sensitivity gradient for the output u(0.5, 0.1) in the same reservoir as in Fig. 1. In the adjoint Eq. (16), the input stimulating the relaxed state takes a form of a delta function at the observation site and time, i.e. $\delta(x-x_i)\delta(t-t_k)$. Accordingly, $\lambda(x, t)$ attains the maximum initially ($t=0.1$), then have gradually abated profiles as time passes backward.

## 3. Sensitivity Behavior

Fig. 3 shows three sensitivity gradient profiles for three outputs u(0.2, 0.1), u(0.5, 0.1), u(0.8, 0.1) respectively under the same reservoir conditions as in Fig. 1. The profile corresponding to u(0.5, 0.1) forms a big peak in the middle and decreases rapidly toward the both ends. This implies that a small variation of the parameter in the middle region where the production well is located has a greater effect on the model output

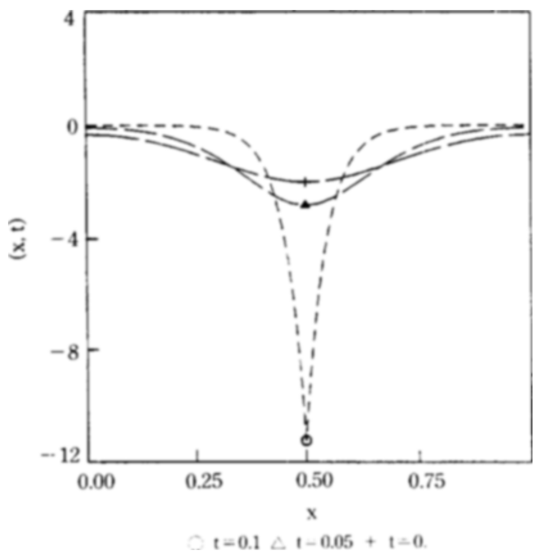


Fig. 2. $\lambda(x, t)$ vs. x for $\partial u(0.5, 0.1)/\partial\alpha$ with $\alpha(x)=1.0$, $n_{pw}=1$ at $x_w=0.5$.

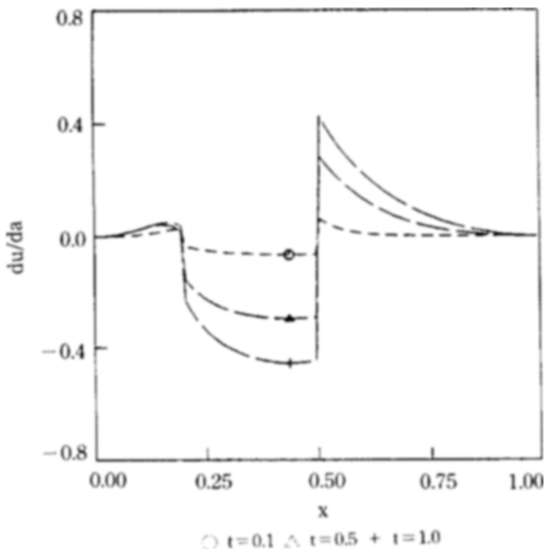


Fig. 4. $\partial u/\partial\alpha$ for $\alpha(x)=1.0$ at x=0.2 with $n_{pw}=1$ at $x_w=0.5$.

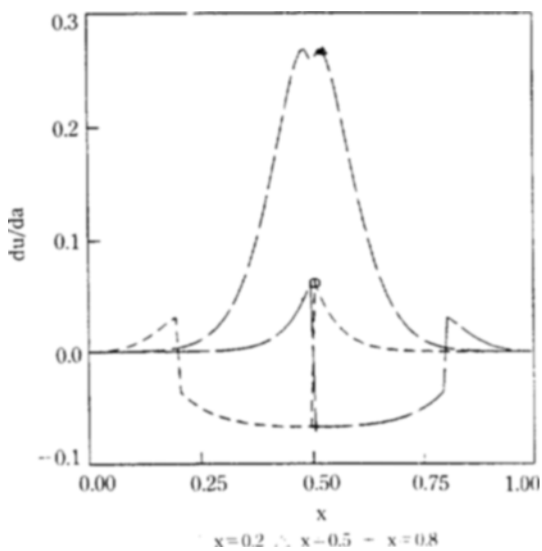


Fig. 3. $\partial u/\partial\alpha$ for $\alpha(x)=1.0$ at t=0.1 with $n_{pw}=1$ at $x_w=0.5$.

u(0.5, 0.1) than variations in the boundary regions have. This can be ascribed to the sharp gradient in the pressure distribution of Fig. 1 near the production well, which will increase the integrand in (17). From a physical view point, the transmissivity $\alpha$ represents a measure of tendency for the system to dissipate its pressure gradient. Thus, in the region where higher gradient builds up, the pressure output has larger sensitivity to the transmissivity.

The major feature of the sensitivity profile for u(0.2, 0.1) is that it has negative values in the region between 0.2 and 0.5. This implies in general that a positive variation of the transmissivity in the region between a specific observation site and a nearby production well will decrease the pressure output at the observation site. It is also observed that parameter variation near the production well also has larger influence than elsewhere. The sensitivity profile for u(0.8, 0.1) exhibits a symmetric profile to the profile for u(0.2, 0.1) which comes from the symmetry of observation sites and reservoir conditions.

A sensitivity profile for a given observation point gets intensified as time passes by. This temporal behavior is well illustrated in Fig. 4 which shows three profiles for the outputs u(0.2, 0.1), u(0.2, 0.5) and u(0.2, 1.0).

Fig. 5 shows sensitivity behavior observed in a reservoir with two production wells located at x=0.3 and 0.7. As in Fig. 3, each profile has peaks around the production wells and negative values between its observation site and production wells. Besides, every profile has zero value at x=0.5, which means that parameter change in the middle of two production wells where the pressure gradient vanishes will have no effect on the output at any point.

Fig. 6 shows sensitivity behavior in a reservoir which has a spatially varying $\alpha(x)=1-0.5\sin 2\pi x$ and a single production well in the middle. Behavior simi-

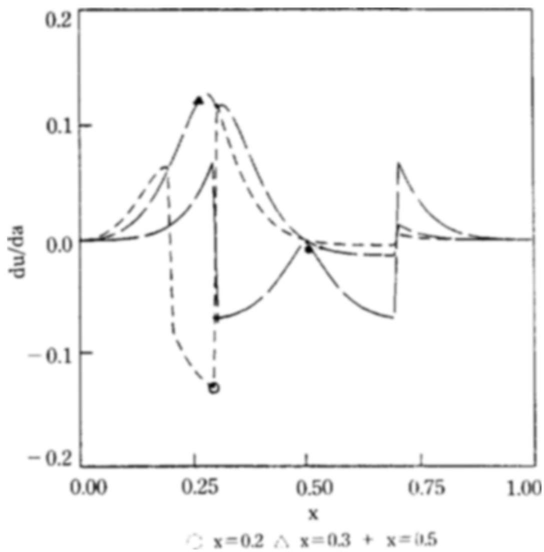


**Fig. 5.** $\partial u/\partial \alpha$ **for** $\alpha(x)=1.0$ **at** $t=0.1$ **with** $n_{pw}=2$ **at** $x_w=0.3,\ 0.7$.

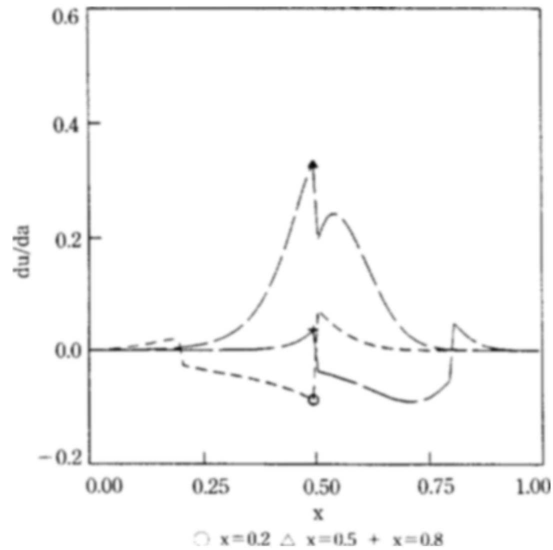


**Fig. 6.** $\partial u/\partial \alpha$ **for** $\alpha(x)=1+0.5\sin 2\pi x$ **at** $t=0.1$ **with** $n_{pw}=1$ **at** $x_w=0.5$.

lar to Fig. 6 is observed on the whole, but symmetry is lacked due to the unsymmetrical shape of $\alpha$. Variation of parameter around $x=0.75$ where the parameter has the minimum value can be seen to have larger sensitivity than around the maximum point $x=0.25$.

The features of sensitivity gradient behavior discussed so far can be summarized as follows.

(1) The pressure output has larger sensitivity to a variation of parameter ($\delta\alpha$) near a production well, while it has very small one to $\delta\alpha$ in the boundary region.

(2) $\delta\alpha$ between an observation well and a nearby production well has a negative effect on the output at the observation well.

(3) $\delta\alpha$ in the middle of two production wells has no influence over any $u_{ik}$.

(4) $\delta\alpha$ in a region where $\alpha$ has larger values has a smaller effect in magnitude, but a more far-reaching range of influence.

(5) As time passes by, the sensitivity gradient has more intense and far-reaching profiles.

### 3. Application to Parameter Estimation

The sensitivity behavior discussed above explains typical error behavior encountered in parameter estimation. For example, large errors of estimates in the boundary region comes from the feature (1) above. This in turn implies that, when judiciously utilized, the sensitivity gradient behavior may provide clues to devising improved estimation algorithms. An illustrative scheme presented below makes use of the fea-

**Table 1. Comparison between uniform mesh and nonuniform mesh schemes**

| Items | Uniform mesh | | | | Nonuniform mesh | | | |
|---|---|---|---|---|---|---|---|---|
| $S^TS$ | 0.0064 | 0.0643 | 0.0229 | 0.0030 | 0.0037 | −0.0041 | 0.0271 | 0.0019 |
| | 0.0643 | 0.6950 | 0.1600 | 0.0229 | −0.0041 | 0.8380 | 0.1520 | 0.0271 |
| | 0.0229 | 0.1600 | 0.6950 | 0.0643 | 0.0271 | 0.1520 | 0.8380 | −0.0041 |
| | 0.0030 | 0.0229 | 0.0643 | 0.0064 | 0.0019 | 0.0271 | −0.0041 | 0.0037 |
| $\det(S^TS)$ | $5.647\times10^{-8}$ | | | | $1.150\times10^{-6}$ | | | |
| Eigenvalues of $S^TS$ | $2.234\times10^{-4}$ | | | | $1.115\times10^{-4}$ | | | |
| | $5.432\times10^{-4}$ | | | | $1.959\times10^{-2}$ | | | |
| | $5.387\times10^{-1}$ | | | | $3.125\times10^{-1}$ | | | |
| | $8.637\times10^{-1}$ | | | | $5.191\times10^{-1}$ | | | |
| $J_{LS}(\alpha)$ | $1.585\times10^{-2}$ | | | | $1.609\times10^{-2}$ | | | |
| Iterations | 12 | | | | 6 | | | |
| $\lVert\hat{\alpha}-\alpha_T\rVert_{L2}$ | $6.236\times10^{-1}$ | | | | $3.005\times10^{-2}$ | | | |

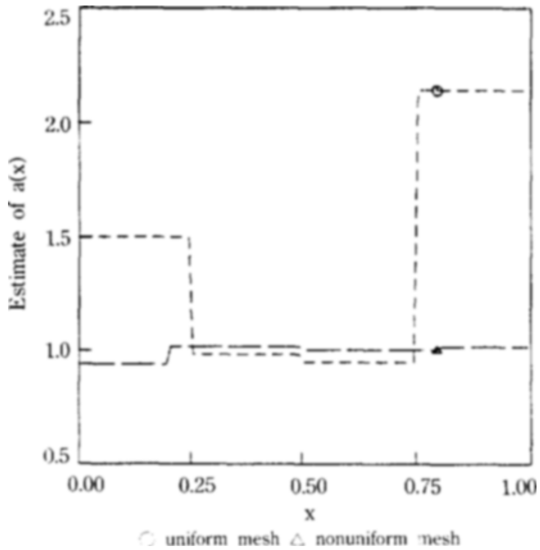


**Fig. 7. Least-squares estimates of $\alpha_T(x)=1.0$: effect of mesh scheme.**

ture (2) to devise a parameter discretization pattern considering the layout of observation and production wells in the domain.

A simple estimation problem was considered where the reservoir has a uniform true parameter $\alpha_T(x)=1.0$ and a single production well in the middle. The observation data consist of simulated pressure measurements obtained 50 times successively with an interval of $t=0.02$ at each observation well located at $x=0.2$, 0.5 and 0.8, respectively. The unknown parameter is represented by four B-splines of order 1, which corresponds to a *zonation* method widely used in reservoir history matching.

When a uniform mesh is applied for B-spline representation on the domain (0, 1) with a breakpoint sequence (0., 0.25, 0.5, 0.75, 1.0), the sensitivity coefficients calculated by (21) will undergo a significant reduction because of the changing sign of sensitivity gradient values around the observation point $x=0.2$ or 0.8 as shown in Fig. 3. This reduction can be prevented by adopting a nonuniform mesh where the breakpoint sequence (0., 0.2, 0.5, 0.8, 1.0) is aligned with the layout of wells.

Table 1 compares the analysis results for two sensitivity coefficient matrices obtained using each mesh scheme. The results of estimation which were started from a rather close initial guess $\alpha^{(0)}=1.1$ are also included in the table. It is clear that the nonuniform mesh yielded the better-conditioned sensitivity coefficient matrix, and hence the more accurate estimate than the uniform mesh. The difference between the two cases is more clearly illustrated in Fig. 7 showing the profiles of estimates. The uniform mesh scheme yielded an estimate with large errors in the boundary region, while the nonuniform mesh scheme led to an estimate with relatively small error over the whole domain.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

| | |
|---|---|
| $B_j$ | : j-th B-spline function |
| d | : descent direction vector |
| H | : Hilbert space |
| J | : objective function or functional |
| L | : length of 1-dimensional reservoir [m] |
| $L_2(\Omega)$ | : space of square integrable functions over $\Omega$ |
| $N_p$ | : number of spline coefficients |
| n | : unit outward normal vector to $\partial\Omega$ |
| $n_{ow}$ | : number of observation wells |
| $n_{pw}$ | : number of production wells |
| $n_t$ | : number of measurements at each observation well |
| q(x, t) | : source term representing withdrawal or injection of fluid [Pa/sec] |
| S | : sensitivity coefficient matrix with respect to spline coefficient |
| T | : total time lapse [sec] |
| t | : time variable [sec] |
| u | : pressure [Pa] |
| $u_0$ | : initial pressure profile [Pa] |
| x | : spatial variable [m] |
| **x** | : spatial variable vector [m] |

### Greek Letters

| | |
|---|---|
| $\alpha$ | : transmissivity of porous medium = permeability/porosity [$m^2$] |
| $\gamma$ | : step size along descent direction |
| $\Delta t$ | : finite difference step size for t |
| $\Delta x$ | : finite difference step size for x |
| $\delta$ | : variation of a function or a functional |
| $\eta$ | : dimensionless factor for source term $=q_iT/u_0$ |
| $\kappa$ | : dimensionless factor for transmissivity $=\alpha_0Tu_0/L^2$ |
| $\lambda$ | : adjoint variable |
| $\mu$ | : viscosity [Pa·sec] |
| $\phi$ | : fluid property term = compressibility/viscosity = [$sec^{-1}$] |

$\varphi$ : functional gradient in a Hilbert space
$\Omega$ : spatial domain of reservoir
$\partial\Omega$ : boundary of spatial domain of reservoir
$\omega$ : vector of B-spline coefficients
$\nabla$ : gradient

### Superscripts

(k) : iteration counter in numerical minimization
(0) : initial guess in numerical minimization
obs : observed value
$\hat{}$ : estimate

### Subscripts

H : Hilbert space
i : point-centered state grid index
m : block-centered parameter grid index
j : B-spline index
k : grid point index for time variable
LS : least-squares
0 : reference value for de-dimensionalization of transmissivity and pressure
ow : observation well
pw : production well
T : true
t : time
t : total
w : production well

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